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Importance of vertical rail track stiffness on dynamic overloading: Limitations of the Eisenmann formulation

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Abstract

Since the development of high-speed rail in Europe in the 1970s, the vertical loads that are transmitted to the rail track by vehicles have been measured. The formulations obtained by Eisenmann and Prud'Homme are the most notable formulations of this period due to their extensive application.

The formula proposed by Eisenmann considered the quality of the rail track and was widely proven for maximum speeds of 200 km/h. With the emergence of high speed trains, Eisenmann proposed a modification to the previously proposed formula to adapt it to the case of high-speed vehicles and lines.

Additionally, the formula of Prud'Homme is important because it introduces new criteria and reveals how the vertical rail track stiffness, the unsprung mass of the vehicle and the quality of the rail track, in addition to the speed of the vehicle, affect the dynamic overloads.

As expressed by this formula, for a given speed and rail track quality, different geotechnical and geometric compositions of infrastructure, which determine the stiffness, cause different dynamic overloads. This fact was not considered in the Eisenmann formulation, exposing its limitations.

The objective of this article is to analyze these limitations; for this analysis, a threedimensional (3D) finite element model of the rail track will be employed to calculate static and dynamic stiffness and obtain the dynamic load values for different types of infrastructure. These results will enable us to analyze the relationship between the strength characteristics of the rail track and the dynamic coefficient C_d, which is understood to be the ratio between the total dynamic loads and the static loads that are transmitted to the rail track.

Keywords: high-speed rail, Eisenmann formula, dynamic coefficient, overload, Prud´Homme formula.

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1. Background

The calculation of the vertical forces on rail tracks is traditionally determined from a static analysis. However, experimental measurements confirmed that the loads that are transmitted to the rail track increased with speed. This finding prompted research in the railway field that proposed the use of a dynamic amplification coefficient C_d.

The expressions of Whinkler and Pihera in 1915, Driessen in 1936 and Schramm in 1955 are the most notable expressions among the first empirical expressions that were proposed to quantify the magnitude of this coefficient.

During an extensive test campaign by Deutsche Bahn (DB) in the 1960s in Germany, the test results indicated that the dispersion of the dynamic loads increased with speed compared with their average. The magnitude of these dispersions was directly related to the quality of the rail track and the vehicle. These results caused Birmann to propose the following formula to the Committee D-71 of the ORE in 1966:

$$C_d = 1 + 0.04 * \left(\frac{v}{100}\right)^3 + a * b * \left(0.1 + 0.01 * \left(\frac{v}{100}\right)^3\right) \qquad (1)$$

In 1969, Eisenmann proved that these dispersions follow a normal distribution, as shown in figure 1.

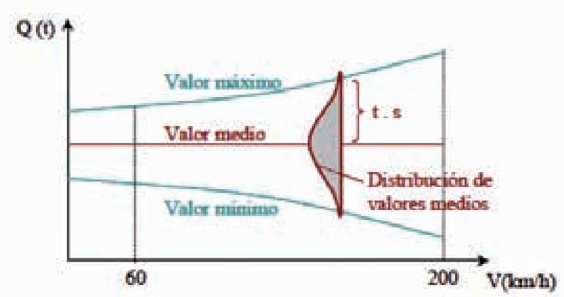

Figure 1: Dynamic oscillation of the load per wheel. Source: Teixeira, 2003

Based on the experimental results, Eisenmann proposed an empirical formula to determine C_d:

$$C_d = 1 + t * s * \varphi \qquad (2)$$

where

t is the factor of probabilistic certainty such that

$t = 1$ 68.3% of the values,

$t = 2$ 95.5% of the values,

$t = 3$ 99.7% of the values;

s is a factor that is dependent on the condition of the rail track,

$s = 0.1$ very good condition,

$s = 0.2$ good condition,

$s = 0.3$ poor condition,

φ is a factor that is dependent on the running speed,

$$\varphi = 1 \qquad \text{for } V \leq 60 \text{ km/h}$$
$$\varphi = 1 + \frac{V-60}{140} \quad \text{for } V > 60 \text{ km/h} \tag{3}$$

Eisenmann's formula considered the quality of the rail track and the confidence interval. This formula has been widely proven for maximum speeds of 200 km/h.

Since the introduction of the high-speed rail in Europe in the 1980s, rail vehicles have been designed to ensure that their loads, which are transmitted to the rail track, are substantially less than the loads transmitted by conventional vehicles. The quality of the newer rail tracks is significantly higher than the quality of existing rail tracks. Consequently, Eisenmann's formula from 1969, which was developed using data from conventional vehicles on conventional lines, was not valid for determining the stresses caused by high-speed trains, such as France's highspeed trains (*Train à Grande Vitesse,* TGV). In 1993, Eisenmann proposed a modification of his previous formula to adapt it to the case of high-speed vehicles and lines. The following expression was defined for the parameter φ for speeds from 201 km/h to 300 km/h:

$$\varphi = 1 + \frac{V-60}{380} \tag{4}$$

In the 1970s, the National Society of French railways (SNCF) developed important theoretical and experimental research to analyze the effect of rail tracks and vehicles on dynamic overloads. This research was performed by Prud'Homme, who used a classic model for modeling a rail track-vehicle system and its behavior and analyzed the excitations produced by the irregularities of a rail track. Prud'Homme applied the theory of random vibrations to develop the well-known formula for calculating the dynamic overloads produced by unsprung masses of a vehicle:

$$\sigma_{(\Delta QNS)=0.45*\frac{V}{100}*b*\sqrt{m_{ns}*K*\gamma(\varepsilon)}} \tag{5}$$

where

$\sigma_{\Delta QNS}$: standard deviation of the dynamic overloads due to unsprung masses,

V: running speed of the vehicle, km/h,

b: variable related to rail track defects and vehicle defects,

m_{NS}: unsprung mass of the vehicle,

K: vertical rail track stiffness, t/mm,

$\gamma(\varepsilon)$: rail track damping

The significance of Prud'Homme's formula is that it introduces new criteria and reveals how the vertical rail track stiffness, the unsprung mass of the vehicle, and the quality of the rail track, in addition to the speed of the vehicle, affect the dynamic overloads.

As expressed by this formula, for a given speed and rail track quality, different geotechnical and geometric compositions of infrastructures, which determines the value of K, produces different dynamic overloads. This fact was not considered by Eisenmann's formula and exposes its limitations. The objective of this article is to analyze these limitations and the relationship between the strength characteristics of the rail track and the dynamic coefficient C_d.

2. Dynamic overloads calculation

2.1 Description of the numerical model

The calculation of the vertical rail track stiffness K was performed using a 3D finite element numerical model of a section of railway rail track using the software ANSYS. One of our objectives is to detect the value of K for the ballasted rail track. The passage of a rail load has been simulated using one model (refer to figure 2). To study the projected ballasted rail track, we have developed a model based on the method proposed in (Gallego, I., 2009) that was used to propose new design criteria in (Gallego, I., 2011) and (Gallego I., 2012 and 2013)

A perfect elastoplastic law was assumed to simulate the behaviors of all materials, with the exception of the rails, elastic pads, sleepers, and the granular material treated with cement and concrete slabs, which are assumed to be governed by an elastic law.

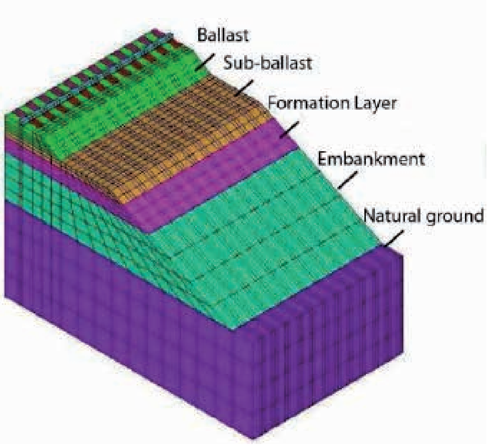

Figure 2. Finite element models for the proposed sections. Source: Gallego et al, 2016.

To obtain the vertical railway rail track stiffness, the following parameters and considerations were used (refer to Gallego, I., (2009) for modeling details):

- The modulus of elasticity EEM of the material that comprises the embankment.
- The height of the embankment hEM.
- Since the natural ground on which the embankment stands along the line consists of rock, with a very high modulus of elasticity, we have assumed that the displacement of this layer is going to be null. Therefore, we can delete it to reduce the computational time to solve the model and achieve similar results.
- The thickness of the ballast and sub-ballast are 35 cm and 30 cm, respectively.
- The well-known rail is UIC 60.
- The elastic pad has k=100 kN/mm.
- The sleeper is the monoblock pre-tensioned AI-99.

To study the influence of these parameters, a series of generic case studies were created with different values (values that are not fixed).

To establish the values that are assigned to the modulus of elasticity of the material that comprises the fill material of the embankment EEM and the other parameters that are needed for the model, the values considered in the soil classification conducted by the UIC are used as a reference. These values are listed in Table 1.

Table 1. Values of the geotechnical parameters considered in the model.

Material	$E(N/m^2)$	ν	$c(N/m^2)$	$\phi(°)$	$\rho(kg/m^3)$
Steel for rails	$2.1x10^{11}$	0.3	-	-	7500
Base plates	$6.91x10^{7}$	-	-	-	-
Sleeper E1	$8.01x10^{10}$	0.25	-	-	2500
Sleeper E2	$5.02x10^{10}$	0.25	-	-	2500
Sleeper E3	$3.69x10^{10}$	0.25	-	-	2500
Ballast	$1.3x10^{8}$	0.2	0	45	1900
Formation layer	$0.8x10^{8}$	0.3	0	35	2000
Material QS1	$12.5x10^{6}$	0.3	10000	10	2000
Material QS2	$25x10^{6}$	0.3	10000	20	2000
Material QS3	$80x10^{6}$	0.3	0.30	30	2000
TGM	$160x10^{6}$	0.25	-	-	2300

Source: Gallego, 2016

EEM values of 12.5, 25 and 80 MPa have been employed. The value of 160 MPa, which corresponds to an embankment that consists of cement-treated granular material (TGM), has also been considered, as in the case of an embankment-structure transition. For the height of the embankment hEM, the values 3, 5, 7, 10 and 15 m have been employed. The considered case studies are determined by the possible combinations between the different values of EEM and hEM. In this study, only the results obtained at 7 m are employed since that height is considered to be average in the existing rail tracks.

The load per wheel considered corresponds to the static load of passenger trains on the Madrid-Seville high-speed line (considering all types of trains); it increases to a value of 186.40 kN per axle.

2.2 Dynamic and static vertical rail track stiffness

From the resolution of each case study, the value for the vertical rail track stiffness K was obtained as follows (refer to Esveld (11):

$$K = \frac{Q}{z}, \qquad (1)$$

where Q is the vertical static load per wheel and z is the summation of all deflections in the vicinity of the load measured on the head of the rail.

The value of K in the Prud'Homme formula is obtained from the dynamic load. Since this load is unknown, iteration is necessary to obtain a converged value of the stiffness based on a certain static load value.

Different types of infrastructures were selected. For each infrastructure, the static stiffness was calculated by entering the static load in the software and consecutively applying it to four sleepers to simulate the passage of an axle. With the static stiffness value obtained, an initial dynamic overload value was calculated. With the total value of the load, the model was recalculated and an initial dynamic stiffness value was obtained. This process was iterated twice; the dynamic rigidity values obtained are listed in Table 2.

Table 2.

Static and dynamic stiffness values (kN/mm) and their ratios.

TYPE OF TRANSITION	STIFFNESS (KN/mm)	SLEEPER 5	SLEEPER 6	SLEEPER 7	SLEEPER 8
Embankment=QS2 Natural ground=QS1	K static	10.800	11.195	10.997	11.263
	K dynamic	12.705	13.023	13.137	13.337
	K dynamic / K static	1.1763	1.1633	1.1976	1.1841
Embankment=QS3 Natural ground=QS1	K static	54.186	54.985	55.476	55.808
	K dynamic	55.956	55.671	57.216	57.073
	K dynamic / K static	1.0326	1.0125	1.0313	1.0226
Embankment=QS2 Natural ground=QS2	K static	16.380	16.658	16.823	16.915
	K dynamic	18.747	19.106	19.335	19.431
	K dynamic / K static	1.1445	1.1469	1.1493	1.1487
Embankment=QS3 Natural ground=QS2	K static	59.936	60.519	60.915	60.519
	K dynamic	61.151	61.735	62.171	62.094
	K dynamic / K static	1.0203	1.0200	1.0206	1.0261

(Source: Gallego, 2012).

From Table 2, we note that

- The dynamic stiffness is always greater than the static stiffness, as expected.
- The difference between the static stiffness and dynamic stiffness decreases as the stiffness value is increased.
- Structures with very elastic infrastructure have 18% greater dynamic stiffness. Rail track structures with a stiffness of approximately 50 and 60 KN/mm have approximately 2% greater dynamic stiffness, which is almost negligible.

Since the minimum appropriate stiffness for high-speed rail infrastructures is 60 KN/mm; the recommended values are 70 and 80 KN/mm. In these cases, the difference between the static stiffness and the dynamic stiffness is very small; use of the simplifying assumption that the static stiffness is equal to the dynamic stiffness seems reasonable.

2.3 Stiffness and dynamic overload results

Due to the nonlinear behavior of the material, the loads need to be applied in several stages. In the first stage, only the weight of the materials is considered until the equilibrium of the stresses is achieved, whereas the load caused by the train is considered in subsequent stages. Because the displacements of interest correspond to the load from the train, they will be obtained by the difference between the totals after applying the train load and the values that correspond to the first stage. According to Committee D-71 of the International Union of Railways Office for Research and Experiments (Office de Recherches et d'Essais de l'Union Internationale des Chemins de fer, ORE, Report No. 28, 1983), the load is distributed on the four sleepers adjacent to the sleeper that is loaded, two on each side. This distribution implies that the real value of the settlement of the rail head caused at a certain point by the load of the wheel that acts on it can only be determined by considering the previous loads that affect this point. The load of at least three consecutive sleepers—the first two sleepers (T5 and T6) previous to the sleeper that is being analyzed (T7), and the latter—must be considered.

Table 3.

Dynamic coefficient values C_d for different types of structure strengths of the rail track.

TYPE OF INFRASTRUCTURE	DYNAMIC STIFFNESS (tn/mm)				DYNAMIC COEFFICIENT			
	K (T5)	K (T6)	K (T7)	K (T8)	SLEEPER 5	SLEEPER 6	SLEEPER 7	SLEEPER 8
Embankment=QS2 Natural Ground=QS1	1.019	0.770	0.674	0.622	1.36	1.33	1.32	1.31
Embankment=QS2 Natural Ground=QS2	1.572	1.371	1.225	1.161	1.41	1.39	1.38	1.37
Embankment=QS2 Natural Ground=QS3	3.323	3.181	3.128	3.061	1.56	1.55	1.54	1.54
Embankment=QS2 Natural Ground=ROCA	3.843	3.781	3.758	3.677	1.59	1.59	1.59	1.58
Embankment=QS3 Natural Ground=QS1	2.890	2.596	2.469	2.393	1.52	1.50	1.49	1.49
Embankment=QS3 Natural Ground=QS2	5.093	4.931	4.905	4.867	1.67	1.66	1.66	1.65
Embankment=QS3 Natural Ground=QS3	7.061	7.061	7.115	7.087	1.77	1.77	1.78	1.78
Embankment=QS3 Natural Ground=ROCA	7.403	7.432	7.471	7.468	1.79	1.79	1.80	1.80
Embankment=MGT Natural Ground=QS1	6.778	6.778	6.853	6.828	1.76	1.76	1.76	1.76
Embankment=MGT Natural Ground=QS2	7.368	7.339	7.397	7.368	1.79	1.79	1.79	1.79
Embankment=MGT Natural Ground=QS3	8.321	8.359	8.434	8.434	1.84	1.84	1.84	1.84
Embankment=MGT Natural Ground=ROCA	8.780	8.776	8.817	8.830	1.86	1.86	1.86	1.86

Source: Prepared by the authors, 2017

From Table 3, we note that the values obtained in sleeper T7 are always similar to the values obtained for the two adjacent sleepers. Three loading conditions did not have to be considered; two conditions would have been sufficient. This finding is explained by the fact that the largest plastic deformation is caused by the loads from the weight of the wheel; it is not caused by the loads from the passage of an axle.

3. Comparison of the numerical model results with the results obtained with the Eisenmann formula.

Figure 3 shows the value of the dynamic coefficient for the two Eisenmann formulations: Values 0.1 and 0.2 were considered for s the infrastructure is in very good condition or good condition. The formulation that is applicable for speeds between 200 and 300 km/h. With regard to the factor of probabilistic certainty t, a value of 3 was employed, which corresponds to the highest statistical reliability.

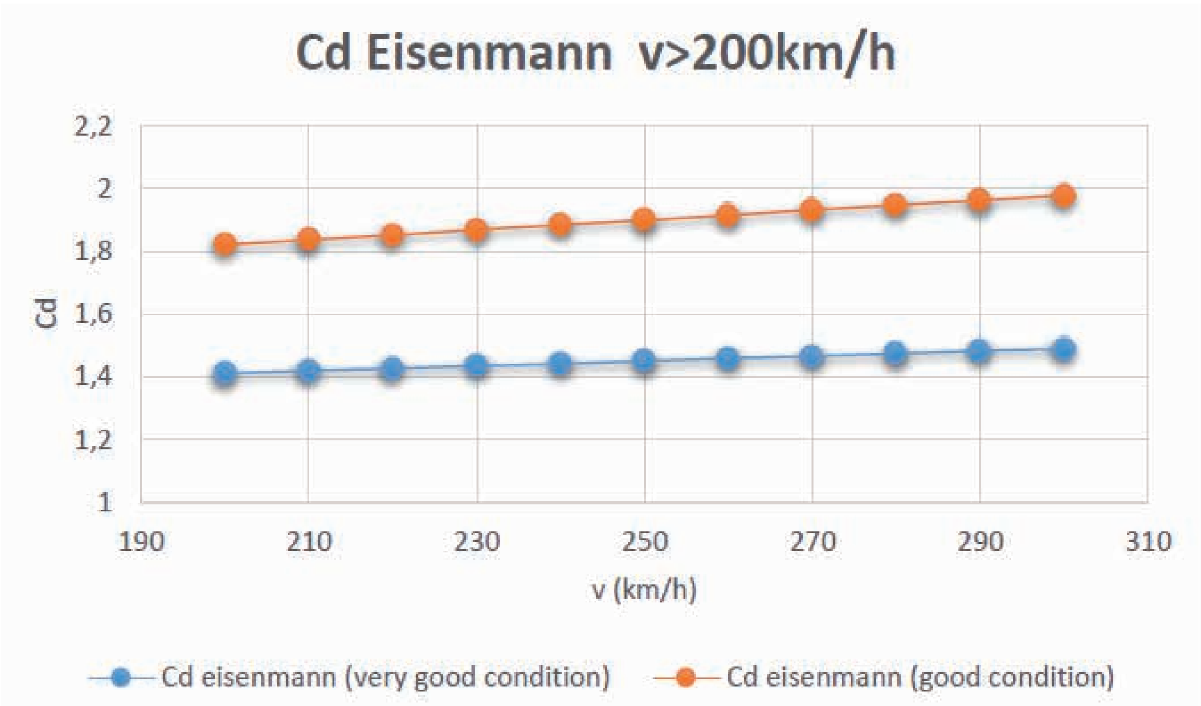

Figure 30. Dynamic coefficient values C_d obtained with the Eisenmann formulations for different conditions. Source: Prepared by the authors, 2017

The limitations of the Eisenmann formula, which does not consider the structure strength of the rail track, are evident when the results obtained by calculating K and using Prud'Homme's formula for the dynamic coefficient are compared with the results obtained by Eisenmann.

For high-speed rail, the recommended minimum stiffness value is 60 KN/mm, and values between 70 and 80 KN/mm are preferred. These values correspond to the last six structures in Table 2.

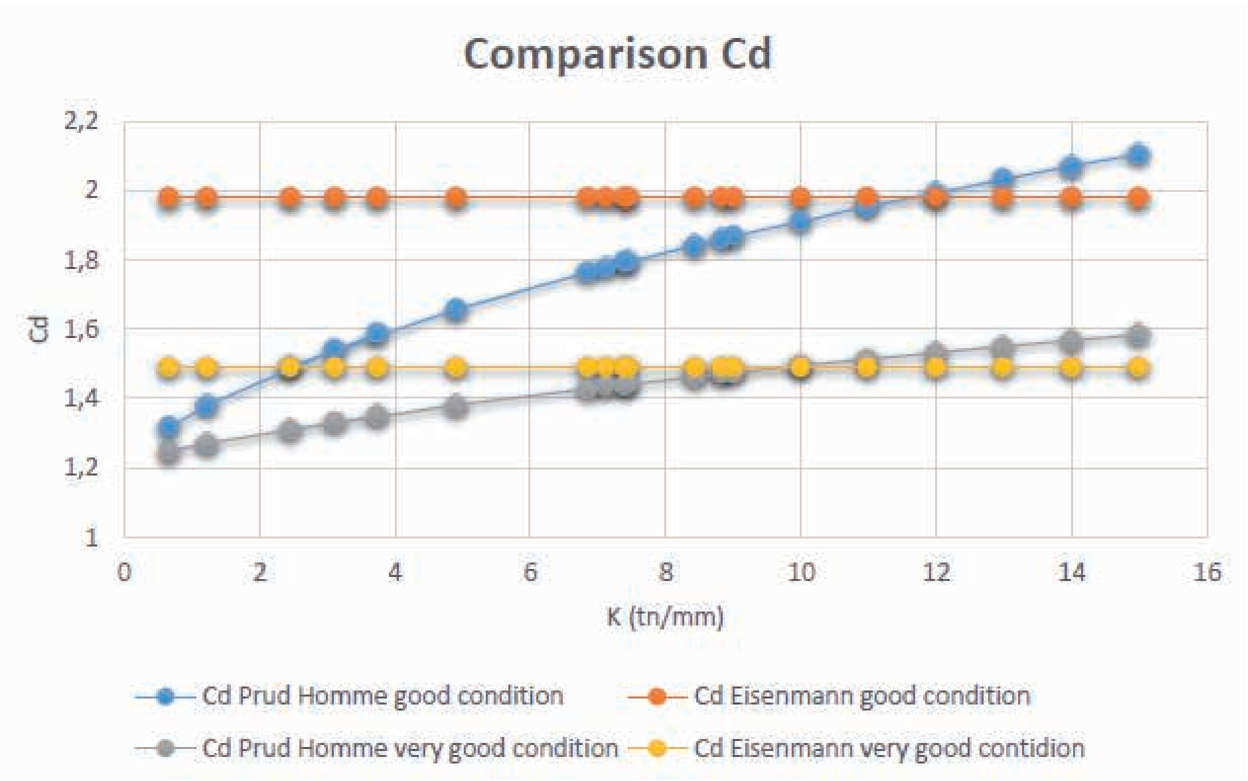

Figure 4. Dynamic coefficient values C_d obtained with the Eisenmann and Prud'Homme formula
Source: Prepared by the authors, 2017

As observed in figure 4 the difference between the coefficient given by Prud'Homme and Eisenmann is smaller as the quality of the high speed track increases.

The difference smaller as it increases the stiffness of the infrastructure, until they converge around the 100 and 120kn/mm where they separate.

Eisenmann offers a superior value to the one of Prud'Homme. In the most flexible regions whereas Prud'Homme occupies the part superior in the most rigid zone.

To determine the proportions on the basis of the criterion of Prud'Homme in the surroundings from the 60 to the 90 kn/mm will help us to rationalize the infrastructure; whereas from the 100 kn/mm when using Eisenmann we would not remain the side of the security.

The values in which Eisenmann approach would correspond to the obtained ones in this paper for the most rigid structures.

However, this infrastructure type is not the one that predominates in the high speed lines, it corresponds to compositions in the zones of transition; reason why to use it is to determine the proportions of the embankment of all the plan would be erroneous.

4. Conclusions

The calculations performed on the 3D numeric model of the rail track proved the following conclusions:

- The dynamic stiffness is always greater than the static stiffness and the difference between them decreases as the static stiffness increases.
- For structures with very elastic infrastructures, the dynamic stiffness is greater than the dynamic stiffness by 18%. However, for rail track structures with rigidities of approximately 50 to 70 KN/mm, the dynamic stiffness is greater by approximately 2%, which is an almost negligible value.
- The stiffness of high-speed infrastructures needs to be greater than 60 KN/mm, and values between 70 and 80 KN/mm are recommended. In these cases, the difference between static stiffness and dynamic stiffness is very small. Therefore, use of the simplifying assumption that the static stiffness is equal to the dynamic stiffness seems reasonable.
- The dynamic amplification coefficient is the result of relating this dynamic loading to the static value of the load. The comparison of the analysis of the coefficient values obtained using Prud'Homme with the analysis obtained using the Eisenmann formula reveals the limitations of the latter formula.
- Is a good approach to use Eisenmann for stiffness superiors to 80 KN/mm, but for more flexible compositions we would be overdesign the infrastructure.
- In addition, the formulas converge in zones of transition where appear the MGT, reason why doesn’t seem suitable to use it to determine the embankment on the rest of the high speed line.

5. References

- Comité D-71, (ORE), (Office de Recherches et d`Essais de l`Union Internationale des Chemins de Fer), (1970), Sollicitation de la voie du ballast et de la plateforme sous l`action des charges roulantes, RP1.
- Comité D-117, (ORE), (Office de Recherches et d`Essais de l`Union Internationale des Chemins de Fer), (1983), Adaptation optimale de la voie classique au trafic de l’avenir, Rapport nº 28 - Abaques de comportement du système voie/fondation-.
- Gallego, I. y López, A. ,2009. Numerical simulation of embankment–structure transition design., Proceedings of the Institution of Mechanical Engineers, PartF: Journal of Rail and Raspid Transit, Vol. 223, pp. 331-343.
- Gallego, I, et al., 2011. Vertical track stiffness as a new parameter involved in designing high speed railway, Journal of Transportation Engineering (ASCE), Vol.137(12), pp. 971-979
- Gallego, I. Heterogeneidad resistente de las vías de Alta Velocidad: Transición terraplénestructura. Editorial Academica Española, 2012

- Gallego, I, et al., 2012. Design of embankment structure transitions for railway infrastructure. Proceedings of the Institution of Civil Engineers Transport, Vol. 165(1), pp. 27-37
- Gallego, I, et al., 2013. Recommendations for numerical rail substructure modeling considering nonlinear elastic behavior, Journal of Transportation Engineering (ASCE), Vol.139(8), pp. 848858
- Gallego, I, et al., 2016. A mixed slab-ballasted track as a means to improve the behaviour of railway infraestructure, Proceedings of the Institution of Mechanical Engineers, PartF: Journal of Rail and Raspid Transit, Vol 230(7), pp. 1659-1672.
- López Pita, A., Infraestructuras Ferroviarias, Ediciones Universidad Politécnica de Cataluña, 2006
- Teixeira, P. F. Contribución a la reducción de los costes de mantenimiento de vías de alta velocidad mediante la optimización de su rigidez vertical. s.l. : Doctoral Thesis, Polytechnic University of Cataluña., 2003.