360.revista de alta velocidad

número 5 - junio - 2018. Pág 263 - 279

Calculation and rational dimensioning of railway infrastructure materials using numerical modelling

Álvarez, Fernando

Balmaseda, Lucía

Gallego, Inmaculada

Rivas , Ana

Sánchez-Cambronero, Santos

University of Castilla-LaMancha (UCLM)[1]

Abstract

In railway engineering, track-vehicle interaction is key to ensuring a certain level of track quality. In order to achieve this quality, it is necessary to carry out an exhaustive control of the deterioration of the railway, this being a consequence of the traffic and the vertical stiffness of the track. In order to obtain an optimum value of stiffness, together with the need to rationalize the sizing of the track, it is useful to study the behaviour of the railway cross-section. The study of this has forced the designers to use different tools, such as numerical models that seek to model the behaviour of the railway platform so that its design is such that the requirements of safety and comfort that are required to the rail transport. This paper shows the realization and proposal of a numerical model that seeks to reproduce as realistically as possible the behaviour of the railway platform, with a dual purpose: to become a calculation tool for the designer and that its use allows the own elaboration of design recommendations.

Keywords: High-Speed railway, Infrastructure, Numerical modelling, Elastic behaviour, Elastoplastic behaviour, Design.

1 *Álvarez, Fernando. University of Castilla-La Mancha (UCLM). Email: Fernando.Alvarez1@alu.uclm.es*
Balmaseda, Lucía. University of Castilla-La Mancha (UCLM). Email: Lucia.balmaseda@uclm.es
Gallego, Inmaculada. University of Castilla-La Mancha (UCLM). Email: Inmaculada.gallego@uclm.es
Rivas, Ana. University of Castilla-La Mancha (UCLM). Email: Ana.rivas@uclm.es

1. Introduction

In recent years, rail transport has experienced great expansion throughout the world, becoming an efficient and competitive transport system for countries with thousands of kilometres of track. In the coming decades, it is envisaged that new and ambitious railway projects will be developed, whose technical difficulties in the interaction with the environment by which it intends to circulate the route and the reach of greater speeds on this one, make it a real challenge for civil engineering; such as the expansion of the High-Speed network in China (Abu Sayeed and Shahin, 2016) and the expected development of the High-Speed line in the United States (Fort and Fort, 2016).

With regard to High-Speed networks, the demographic and economic growth of many countries, which have such a rail network with optimum operating conditions and study experience, have led to higher speeds in order to comply the demand and / or economic objectives required of this transportation system. The increase of the speed could generate an increase in the values of the loads that are transmitted to the railway platform, being the speed proportional to the value of this one. This is related to the effect of the weight of the non-suspended masses of the train, since, if this weight does not change but increases the speed value, the dynamic effects that occur in the vehicle-track interaction, generates a dynamic overload that increases the values of the loads to be supported by the track. If we now refer to rail freight transport, future commercial expansion of several countries may lead to increased freight and / or transport capacity by trains, either by increasing the number of cars or the axle load being transmitted to the railway platform (Li, et.al., 2016). These increases in the magnitudes of the loads show that, if it is desired that this transport system correctly fulfils its function, the train must circulate by a means that guarantees safety and stability to the infrastructure and comfort for the passengers and loads to be transported.

On the other hand, awareness of the economic and landscape impact generated by the design and construction of a High-Speed lines has led to an awareness of the importance of valuing and treating the local materials of the trace, avoiding the massive waste of these through their disposal can consider the possibility of subjecting them to a treatment that improves their geomechanical behaviour (Gomes Correia, et.al., 2016), and make it fit to be usable in the construction of the railway platform.

So far, the sizing of the railway platforms has been carried out from the point of view of the experience obtained in the realization of projects, being in the case of the High-Speed, a very conservative sizing for the seat layers that form the railway platform. In the current context of economic crisis, it is necessary, in the field of exploitation and design, to introduce certain design criteria so that the cross-section of the railway is defined according to criteria of efficiency and rationalization of the same.

These three aspects; increase of the loads that request the railway platform, geomechanical characterization of recycled materials that form it, and introduction of new criteria that realize a rational and efficient design of the platform; are the key reasons for carrying out a study that has, as a final objective, the design of the railway cross-section in accordance with criteria of technical efficiency, being stable and functional in the face of high load conditions and economic efficiency, avoiding an oversizing of the same .

The purpose of this paper is to describe the elaboration of design recommendations, for which it has previously been necessary to make a numerical model from the refined other existing model, and can also serve as a future phase in the performance of a dynamic analysis.

Finally, the structure of the following article is described, being organized as follows:

First, the description the methodology adopted in the realization of said numerical model, followed by a description of the characteristics and cases of analysis considered in the mime; then the main results obtained from the resolution of the calculation by the numerical model will be presented, being these mainly values of seats and vertical tensions in the different layers.

2. Methodology

The study of the geotechnical behaviour of the railway platform is one of the most difficult problems in the field of civil engineering in terms of its resolution and interpretation of the results obtained. From the outset, this problem has been approached through the use of classical analytical solutions obtained from the use of Elasticity Theory hypotheses (Winkler, 1867; Poulos and Davis, 1974; Jimenez Salas, 1981) until the and the development of new mathematical and numerical tools (Indraratna, 2016), such as the Finite Element Method (MEF) and Discrete Elements Method (MED) (McWilliams, et.al., 2000; Huang and Tutumluer, 2011) has made it possible to solve this problem in a more precise way, allowing the addition of new variables that try to reproduce more accurately the behaviour of the railway platform.

One of the effects that computers and numerical methods had on the calculation of railway platforms was the possibility of modelling the behaviour of the materials of the platform in its elastic or plastic form together, thus integrating the model of elastoplastic behaviour; On the other hand, it was also improved the capacity of calculation before cases of loads that considered the variation in the space and time of this one, giving place to the dynamic analyses of load in railway platforms with a wide development in our days. However, these two aspects are not yet fully integrated into current numerical models, since the use of the Plasticity Theory hypotheses requires the study of complex constitutive laws and the use of additional mechanical parameters that are not required if It is assumed an elastic behaviour of the materials and therefore, many current numerical models still assume elastic or derivative behaviour models as the hyperbolic model (Gallego, et al., 2013; Shih, et.al., 2017). With respect to dynamic analysis, most of the commercial software that operate with the use of finite elements have tools that allow the modelling of mobile load cases and dynamic properties of materials, thus having a greater difficulty in adequately characterizing the geomechanical behaviour of these.

The numerical model presented here allows to model the geometry and the elements that form the section of railway platform used in the Spanish High-Speed. Different recommendations and modelling methodologies used by different have been used for its elaboration, being the most outstanding recommendations in normative series (Ministerio de Fomento, 1999) and, as numerical models, those made by Gallego (2012) and others (Gallego, et.al., 2013).

With the orientation of these existing models, it has been refined or improved in the same as will be seen, by eliminating the ballast material that confines, in the transverse and longitudinal direction, the sleepers of the platform, generating a model with confining ballast and without it. In addition to this, a deepening in the interpretation of the results that obtain numerical resolution is made, comparing these assuming both a constitutive model of elastic behaviour and elastoplastic for the granular materials that form the seat layers of the platform, all for a simple static load case.

Finally, for the calculation of the numerical model and the interpretation of the results obtained, a sensitivity analysis was carried out by varying the value in one of the resistant parameters of the granular materials that form the railway platform, these being in particular the Formation layer and Subballast.

3. Numerical model

3.1 Description

The numerical model presented here consists of a three-dimensional Finite Element model and fully parameterized, that is, it allows the user to directly modify the geometric properties that define the railway track section as well as the mechanical and geomechanical characteristics that define the behaviour of the elements of the superstructure of the track and the granular layers of the infrastructure. The tool used for the numerical modelling is commercial software ANSYS® Mechanical APDL 17.2, which requires the use of external programming codes or through the direct use of the interface.

With regard to the same, only the most relevant elements of the superstructure are modelled: rail, bearing plate, sleepers and ballast; and infrastructure layers: subballast, formation layer and embankment or subgrade. In this way, the finite element model acquires the form shown in Fig. 1, simulating half of the section of track as a single track.

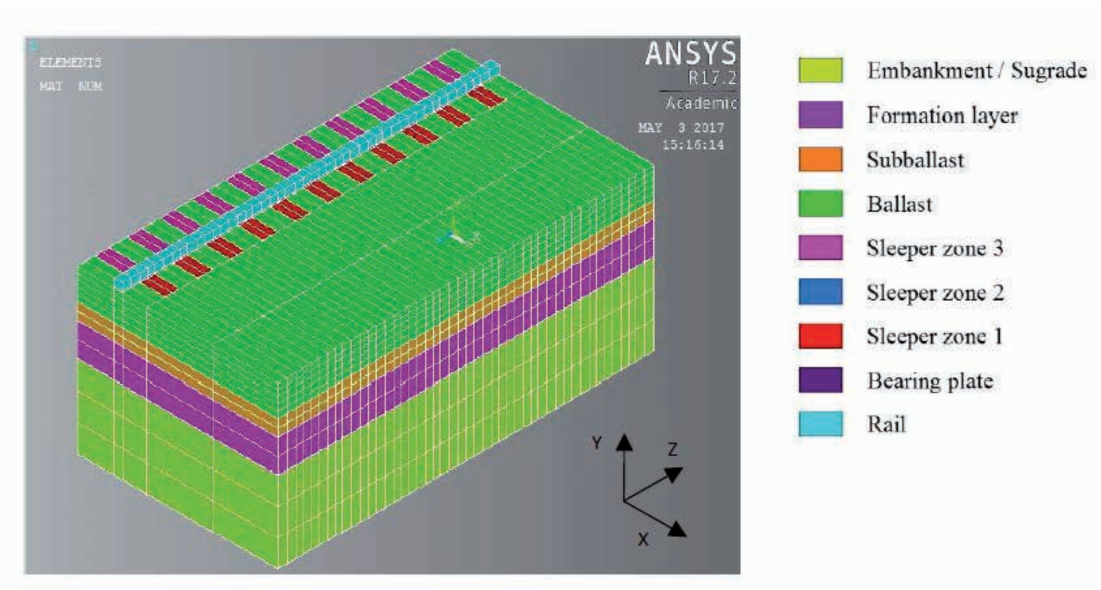

Fig. 1. General view of the numerical model and its mesh.

3.2 Geometry and material properties

For the definition of the geometry and the elements that form the section of railway platform have been used the different recommendations and methodologies of modelling collected by the different mentioned above.

With regard to the elements of the track superstructure: UIC60 rail, bearing plate and sleepers; These have been modelled with equivalent geometries of parallelepiped volumes by threedimensional elements of 20 nodes and that, together with the adjustment of the mechanical properties of materials (Gallego, 2012), it is possible to model with more accuracy the structural behaviour of these in the different situations of load.

In relation to the seat layers, these are re-modelled with parallelepiped volumes assuming that each of these behaves as a continuous medium. Further:

1. It has been assumed that all granular materials of the granular layers are homogeneous and isotropic materials where the geomechanical properties are constant throughout the volume and there is no change in the value of these.

2. In the case of ballast, by its rheology and interaction between particles, it would be more advisable to use discrete elements for its numerical modelling, in this numerical model it is modelled as a continuous medium by finite elements, paying close attention to that, if a model is adopted sufficiently adequate, it will be able to reproduce a more accurate tenso-deformational response than using simpler models (Ishikawa, et.al., 2014).

3. The geometry of the cross section has been defined following the dimensions proposed by ADIF (2006) in its different regulations for High-Speed lines, as shown in Fig. 2. In this figure you can see how it would also be configured the section of eliminate the confining ballast by refining the numerical model.

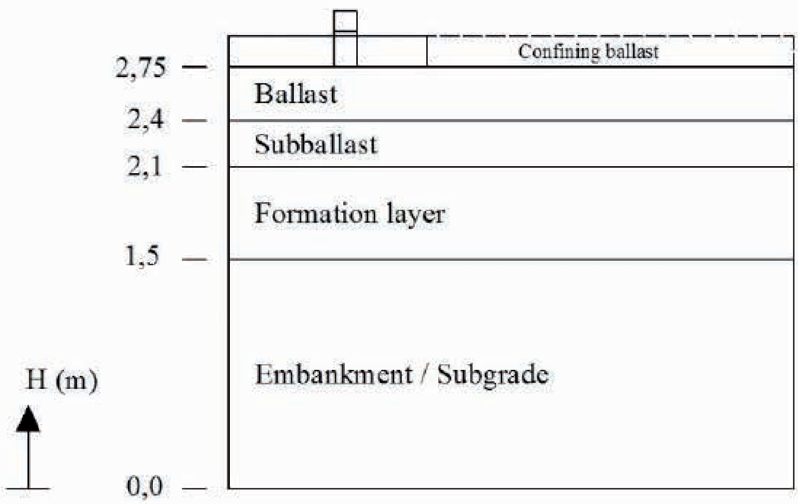

Fig. 2. Layer dimensions for the High-Speed cross section in Spain.

The mechanical and geomechanical properties of the different materials are listed in Table 1. In this it can be seen how, for the sensitivity analysis, the value of the internal friction angle has different values for two of the granular materials shown. In the case of the Formation layer there will be a decrease in the resistance and, in the Subballast the opposite. It has been assumed that all granular materials are in perfectly drained conditions, so that the tensions obtained will be in terms of effective coincident with the values of total stresses.

Table 1.	E (Pa)	v	ρ (N/m³)	c (Pa)	Φ (°)
Rail (Steel)	2.10E+11	0.3	7500	-	-
Bearing plate	6.91E+07	0.4	1	-	-
Sleeper zone 1 (Concrete)	7.83E+10	0.25	2500	-	-
Sleeper zone 2 (Concrete)	4.90E+10	0.25	2500	-	-
Sleeper zone 3 (Concrete)	3.59E+10	0.25	2500	-	-
Ballast	1.30E+08	0.2	1900	0	45
Subballast	1.20E+08	0.3	1900	0	Φ_{SB1}, Φ_{SB2}
Formation layer	8.00E+07	0.3	2000	0	Φ_{FL1}, Φ_{FL2}
Embankment/ Sugrade	2.50E+07	0.3	2000	1.00E+04	20

3.3 Treatment of interfaces

To better model the behaviour and interaction of the different elements, especially the contact between the sleeper and the ballast, a numerical tool has been used to solve local problems in contact areas, such as the high concentration of stresses due to the existence of two materials with very different stiffness. The tool used in such software is to equalize the displacements by duplicating a node for both sides of the interface through the coupling and nodes (ANSYS, 2008), which allows to solve satisfactorily the problem of the discontinuity in the stresses and deformations and also, does not raise the computational cost in the resolution of the model.

3.4 Analysis cases

With respect to the behavioural laws that govern the behaviour of the materials, we have chosen to model and calculate the model assuming two ways: one in which it considers that all the materials of the railway platform are governed by a linear elastic behaviour and another in which, the granular materials of the platform are governed by a non-linear behaviour of elastoplastic type.

For the elastoplastic behaviour model, a perfect plasticity model has been assumed (ANSYS, 2013), where the hardening and softening effects of the material defined by the hardening parameter H' are not considered in the following expression:

$$E^{ep} = E \cdot \frac{H'}{E + H'} \tag{1}$$

where E^{ep} is the elastoplastic deformation modulus and E the Young's modulus. If we consider a null value for H', the stress-strain curve will have the form shown in Fig. 3 (Oliver and Agelet, 2002; Chaves, 2013), in which it is represented that when the material reaches the stress threshold value σ_e, its tensional states will only move within the horizontal elastoplastic branch, but if it is not reached, it will continue to behave as a linear elastic material.

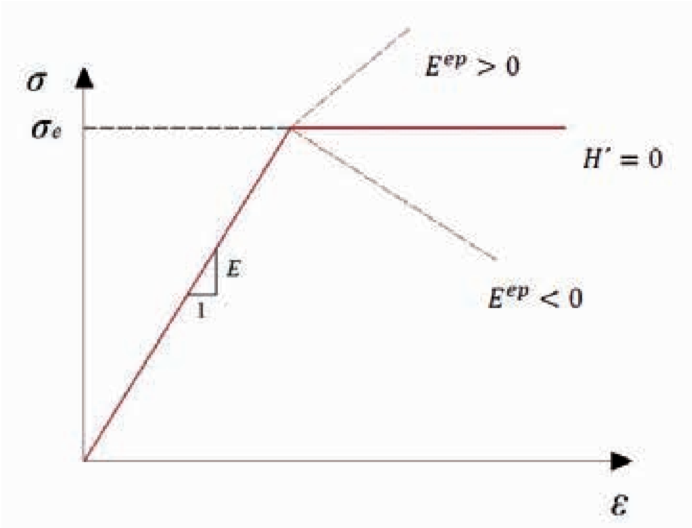

Fig. 3. Elastoplastic model assuming perfect plasticity.

For the elastoplastic model, a rupture criterion o has also to be used, namely the Drucker-Prager criterion, a generalized version of Von Mises's perfect plasticity model and also a smoothed approximation of the Mohr-Coulomb model (Jiménez Salas, 1981; Chaves, 2013), see Fig. 4.

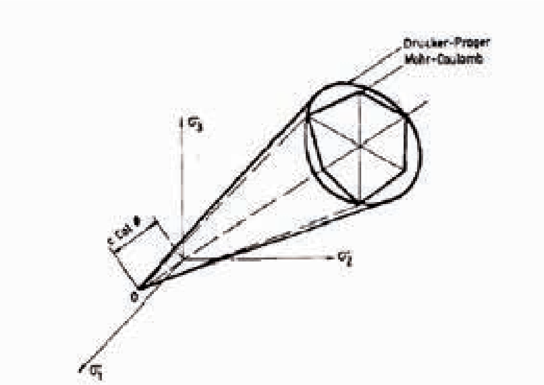

Fig. 4. Conceptual representation of the creep surface for the frictional elastoplastic models of Mohr Coulomb and Drucker-Prager (Gens and Potts, 1988). Note: The correct value of c·cotf is c $\sqrt{3}$ cotf (Chaves, 2013).

The load to be applied in the numerical model consists of a static load obtained from the weight of the load values per axis corresponding to Spanish train models used in High-Speed, which reaches an equivalent value of 186.4 kN per axis (Gallego, 2012). With respect to the purely elastic model, only one loading state corresponding to the application of the load □ on the platform will be applied, whereas for the elastoplastic model, depending on the tensions and deformations of the load history, three states of load: one corresponding to the self-weight (LS1), another one due to the application of the load Q (LS2) as in the elastic model and, last one where the discharge (LS3) is considered.

With all these aspects, the models under study have been elaborated according to the following criteria:

1. All models will maintain the constitutive model of elastic behaviour for the Rail, Bearing plates and Sleepers due to their high stiffness.
2. The comparison between elastic and elastoplastic behaviour models will only refer to granular materials of the railway section.
3. The sensitivity analysis will be performed by varying the value of the internal friction angle corresponding to the Subballast and the Formation layer.

Table 2 shows, in summary, all the models or cases of analysis to be analysed, according to the behaviour of the granular materials, the existence or not of confining ballast and the episodes of load to be considered.

Table 2. Summary of characteristics of the models to be studied.

Model	Constitutive model-Infrastructure later	Confining ballast	Load step
#1	Elastic	Yes	LS2
#2	Elastic	No	LS2
#3	Elastoplastic (Φ_{FL1}, Φ_{SB1})	Yes	LS1+LS2+LS3
#4	Elastoplastic (Φ_{FL1},Φ_{SB1})	No	LS1+LS2+LS3
#5	Elastoplastic (Φ_{FL2}, Φ_{SB1})	Yes	LS1+LS2+LS3
#6	Elastoplastic (Φ_{FL2}, Φ_{SB1})	No	LS1+LS2+LS3
#7	Elastoplastic (Φ_{FL1}, Φ_{SB2})	No	LS1+LS2+LS3
#8	Elastoplastic (Φ_{FL2}, Φ_{SB2})	No	LS1+LS2+LS3

4. Results

4.1 Adjustment model

During the resolution of the numerical model that considered the linear elastic behaviour of all the materials of the platform, it was detected that the vertical compression stresses in the ballast under the loaded sleeper reach values well below the usual ones of 100 to 120 kPa, obtained in experimental observations performed for High-Speed sections in Spain (Gallego, et. al., 2013). This could be because, considering the ballast as a linear elastic material, makes the ballast elements located above the support plane of the crossbeam oppose the compressions that occur in said plane, generating tensions that They decrease the value of the compressions that are given in the sleeper-ballast interface. To solve the problem we defined the following strategies to consider for the numerical model:

1. Eliminate the ballast that confines the sleepers in the longitudinal and transverse direction of the track, see Fig. 2.
2. Assume in the ballast a law of elastoplastic behaviour, without cohesion, to avoid the appearance of tractions in the same.

The second of the strategies did not make sense since it was contemplated to compare the elastic model with the elastoplastic. Therefore, it was decided to apply the first of them to check if really, removing the top layer would influence the vertical stresses that occurred in the model. Fig. 5 shows the different steps that were performed in such a strategy, where compression stresses has negative sign and tractions stresses have positive sign: Fig. 5a shows the platform model with confining ballast that assumes linear elastic behaviour for all materials, being able to observe traction stresses in the area next to the sleeper where the load is applied and the adjacent ones, above the support plane with the ballast, and in areas close to the limits in the numerical model domain; If we eliminate the confining ballast and continue assuming an elastic behaviour, Fig. 5b, the traction stresses in the area next to the loaded sleeper disappear above the support plane of the sleeper, where they no longer oppose the compressions and they increase their value below the loaded sleeper but, in the lower and top zone to the support plane of the adjacent sleepers, the tractions continue to persist and continue to the limit of the model, possibly by a “distortion” of the numerical model; Considering now to assume an elastoplastic behaviour for the granular materials together with the presence of confining ballast in the sleepers, we have what is shown in Fig. 5c, where now next to the loaded sleeper, despite having considered a cohesion value (c = 0), tractions appear above the contact plane of the sleeper, possibly due to the ballast that confines the sleepers, whereas in the adjacent sleepers, the tractions persist below and above the contact plane due to arrange nodes coupled to the cross-ballast contact and are eliminated completely in the limits of the model when considering a value of cohesion null for the ballast; Finally, the elimination of the confining ballast now results in what is shown in Fig. 5d, where now the tractions disappear completely next to the sleeper where the load is applied and in the limits of the model, nevertheless they continue present In the plane of contact of the sleepers adjacent to the loaded one due to the coupled nodes.

In view of these results, the following can be stated:

1. The occurrence of tractions alongside the loaded sleeper, above the support plane, is present in all cases where confining ballast is available in the sleepers, decreasing the value of the compressions in the underlying layers. These disappear by mistrusting the sleepers in all cases.
2. The occurrence of tractions under the contact plane in the sleepers adjacent to the loaded sleeper is a consequence of the use of the coupled nodes. Considering a zero cohesion for the ballast and that there is no ballast that confines the sleepers, makes them decrease in the elastoplastic models with respect to the elastics.
3. The tractions in the limits of the numerical model are due to a "distortion" of the results as a result of finite element modelling and, they disappear only when considering null cohesion in the ballast.

Based on these observations, it was decided to analyse and compare the results obtained by the elastic and elastoplastic models that better simulated the tenso-deformational behaviour, which were the ones that did not have confining ballast, Models #2, #4, #6, #7 and #8 defined in Table 1, remaining the others discarded when not fulfilling this condition.

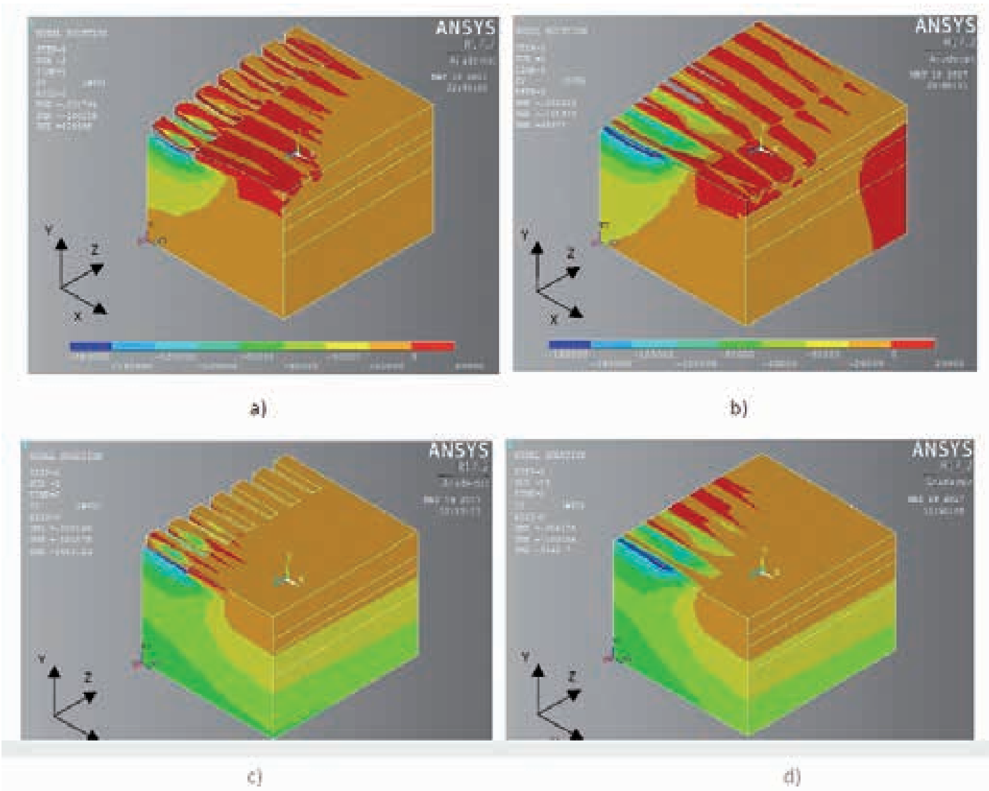

Fig. 5. Evolution of the numerical model readjustment. (a) Elastic model with confining ballast; (b) Elastic model without confining ballast; (c) Elastoplastic model with confining ballast; (d) Elastoplastic model without confining ballast. Values in Pa.

4.2 Compressions in each layer

Table 3 shows the vertical compressive values below the sleeper where the load is applied, for each of the layers of the model assuming a linear elastic behaviour for all materials (Model #2), generating a total seat δ of 2.38 mm obtained from the sum of the compressions in each layer ρ_i, such that according to Fig. 6:

$$\delta = \sum_{i=1}^{n} \rho_i \qquad (2)$$

Table 3. Compressions in each assembly and layer in the elastic model (Model #2).

	ρ_i (mm)	% respect to δ
Rail-Bearing plate-Sleeper	-0.48	20.04 %
Ballast	-0.24	10.22 %
Subballast	-0.15	6.16 %
Formation layer	-0.31	12.97 %
Embankment/Subgrade	-1.21	50.61 %

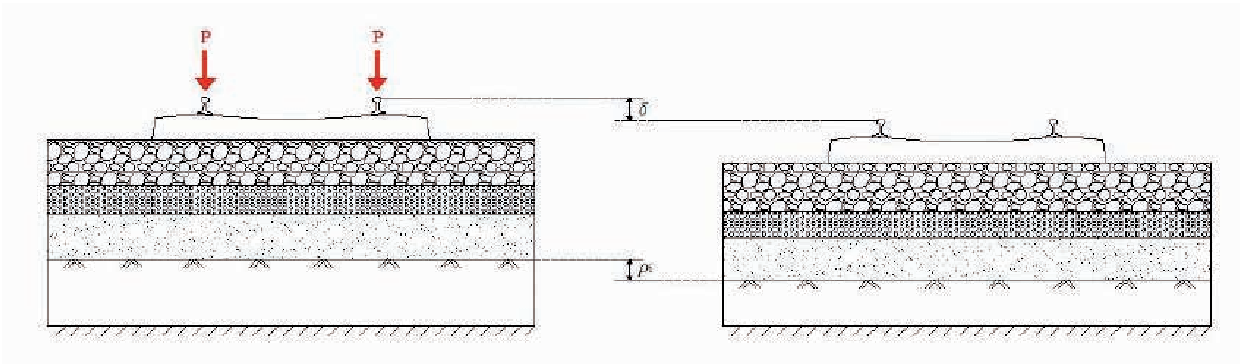

Fig. 6. Vertical settlement of the track and compression of each layer.

As expected, the Embankment or Subgrade that serves as the base to the rest of layers and elements, is the layer that has greater compression due to its smaller modulus of deformation compared to the rest of layers. After this, the Rail-Sleeper-Bearing plate assembly is the one with the greatest seat, because the seat plate has a much smaller modulus of elasticity than those corresponding to the rail and sleeper.

If we now compare the compression values in each of the layers that form the railway platform for all the elastoplastic models studied, see Table 4, we see that there is a redistribution of the values of vertical compressions and in the percentages with respect to the total seat. The RailSleeper-Bearing plate assembly and Ballast, even improving the Subballast resistance or decreasing that of the Formation layer, continue to have the same behaviour when returning to have the same values of compressions. As for the Subballast, we see that clearly, varying its angle of friction does not influence significantly since the values of compressions between the four elastoplastic models are very similar.

One of the main differences if we compare the results between the elastic model and the elastoplastics is to see that the settlement values in the elastoplastics are higher. This is due to the consideration of the sum of the elastic and plastic components that can now be given, which causes the total deformation to be defined by:

$$\varepsilon = \varepsilon^{e} + \varepsilon^{p} \qquad (3)$$

where ^{e} is the elastic deformation of the element and ^{p} the plastic deformation of the element. This is important to consider since the railroad is subject to numerous load cycles during its useful life, which can result in an increase of the plastic or irreversible deformation component and generate permanent deformations that can compromise the safety of the track.

Table 4. Compressions and vertical settlement in each assembly and layer in the each elastoplastic model.

	Model #4 (Φ_{FL1} = 35° ; Φ_{SB1} = 35°)		Model #6 (Φ_{FL2} = 20° ; Φ_{SB1} = 35°)	
	ρ_i (mm)	% respect to δ_1	ρ_i (mm)	% respect to δ_2
Rail-Bearing plate-Sleeper	-0.46	18.11 %	-0.45	16.61 %
Ballast	-0.24	9.45 %	-0.22	8.12 %
Subballast	-0.16	6.30 %	-0.17	6.27 %
Formation layer	-0.34	13.39 %	-0.48	17.71 %
Embankment/Subgrade	-1.34	52.76 %	-1.40	51.66 %
	Model #7 (Φ_{FL1} = 35° ;Φ_{SB2}= 45°)		Model #8 (Φ_{FL2} = 20° ; Φ_{SB2} = 45°)	
	ρ_i (mm)	% respect to δ_1	ρ_i (mm)	% respect to δ_4
Rail-Bearing plate-Sleeper	-0.46	18.25 %	-0.45	16.79 %
Ballast	-0.24	9.52 %	-0.22	8.21 %
Subballast	-0.15	5.95 %	-0.16	5.97 %
Formation layer	-0.34	13.49 %	-0.47	17.54 %
Embankment/Subgrade	-1.33	52.78 %	-1.39	51.87 %

δ_1 = 2.54 mm in Model #4

δ_2 = 2.71 mm in Model #6

δ_3 = 2.52 mm in Model #7

δ_4 = 2.68 mm in Model #8

The vertical settlement of the track is a parameter that allows the calculation of the vertical stiffness of the track K, defined by the following expression (López Pita. 2001):

$$\mathrm{K} = \frac{Q}{\delta} \tag{4}$$

This parameter is important to consider by the designer in any project of design and maintenance of the railway platform as this function of the mechanical properties that occur in the elements of the superstructure and layers of railway infrastructure. A low vertical stiffness value would mean having a flexible path with little dissipation energy and a bending deformation of the ballast that would generate its degradation by abrasion whereas, a high value of vertical stiffness would lead to increase the dynamic force at the wheel-track interface, transmitting a greater load to the sleeper-ballast interface and may cause deterioration and fatigue of the track elements (Woodward, et.al., 2014). Table 5 shows the vertical stiffness values obtained by applying the above expression for the value of the load defined in this analysis, in which we can see how the elastic model (Model #2) has a higher value than the others. For the fact of having a lower seat value with respect to the others, leading to a value greater than could occur in reality if we assume that an elastoplastic model reproduces reality better.

Table 5. Vertical stiffness values for each studied model.			
Model	Pointed load (kN)	Vertical settlement δ (mm)	Vertical stiffness K (kN/mm)
#2	186.4	2.38	78.32
#4	186.4	2.54	73.39
#6	186.4	2.71	68.78
#7	186.4	2.52	73.88
#8	186.4	2.68	69.43

4.3 Vertical stresses in each layer

After calculating the displacements for the elements of the model, the values of the stresses in them can be calculated from the following constitutive relation (Chaves, 2013):

$$\{\sigma|(x,y,z)\} = [\mathrm{D}]\{\varepsilon\,(x,y,z)\} \tag{5}$$

where $\{\sigma\}$ is the stress vector, { } is the strain vector and $[\mathrm{D}]$ is the constitutive matrix which contain the mechanical properties of the material. In the case that we consider a calculation with elastic behaviour of the material, matrix $[\mathrm{D}]$ will only consider this behaviour model through the parameters that define it; On the other hand, if we consider an elastoplastic behaviour, this matrix is modified by considering the plastic contribution in the calculation of the solution, which according to Jiménez Salas (1980), the new constitutive matrix $[\mathrm{D}']$ is given as the difference:

$$[\mathrm{D}'] = [\mathrm{D}^{e}] - [\mathrm{D}^{p}] \tag{6}$$

where $[\mathrm{D}^{e}]$ is the elastic constitutive matrix, $[\mathrm{D}^{p}]$ is the plastic constitutive matrix, and $[\mathrm{D}']$ is the elastoplastic constitutive matrix. The different calculation methodology between the elastic and elastoplastic analysis has been seen in a significant way in the calculation of the previous displacements. This difference will also affect the calculation of tensions according to the type of calculation that is used according to the type of behaviour defined for the material.

From the point of view of design, the knowledge and determination of the profile of vertical stresses in the railway section is important to know the values that are achieved in the different materials of the section. These values of stresses obtained by the calculation allow us to see if the material reaches the maximum admissible stress, obtained experimentally, that if reached, could give rise to some of the most common geotechnical problems in railways (Li, et.al., 2016), possibly due to an inefficient dimensioning of the section or the use of inappropriate materials

Fig. 7 shows the profiles of vertical stresses that develop throughout the entire height of the railway platform below the sleeper where the load Q is applied.

1. Firstly, it is verified that by making the Subballast stiffen by increasing the value of its friction angle □ (Models #7 and #8), it leads to a greater stress absorption and, therefore, the stress profile moves to the right, towards higher stress values.
2. With regard to the Formation layer:
   - If it has a friction angle of 35° constant and the Subballast of 35° (Model #4) and it changes to 45° (Model #7), the differences in the vertical stress level are negligible.
   - In the same way, it happens in the case of having a constant value 20° in the Formation layer of form and to re-vary that of the Subballast (Models #6 and #8).
3. On the other hand, a constant value of the internal friction angle of 35° in the Subballast and with a variation in the Formation layer (Models #4 and #6) makes a significant influence of vertical stress difference in the layers of Ballast and Subballast. The same occurs when a constant value of friction angle is set in the Subballast of 45° and the shape layer (Models #7 and #8) varies with the same difference of vertical stresses in the two upper layers.
4. In the contact Formation layer-Embankment at the height of 1.5 m. it is observed that the vertical stresses are equal for all elastoplastic models until reaching the 0 or base of the model.

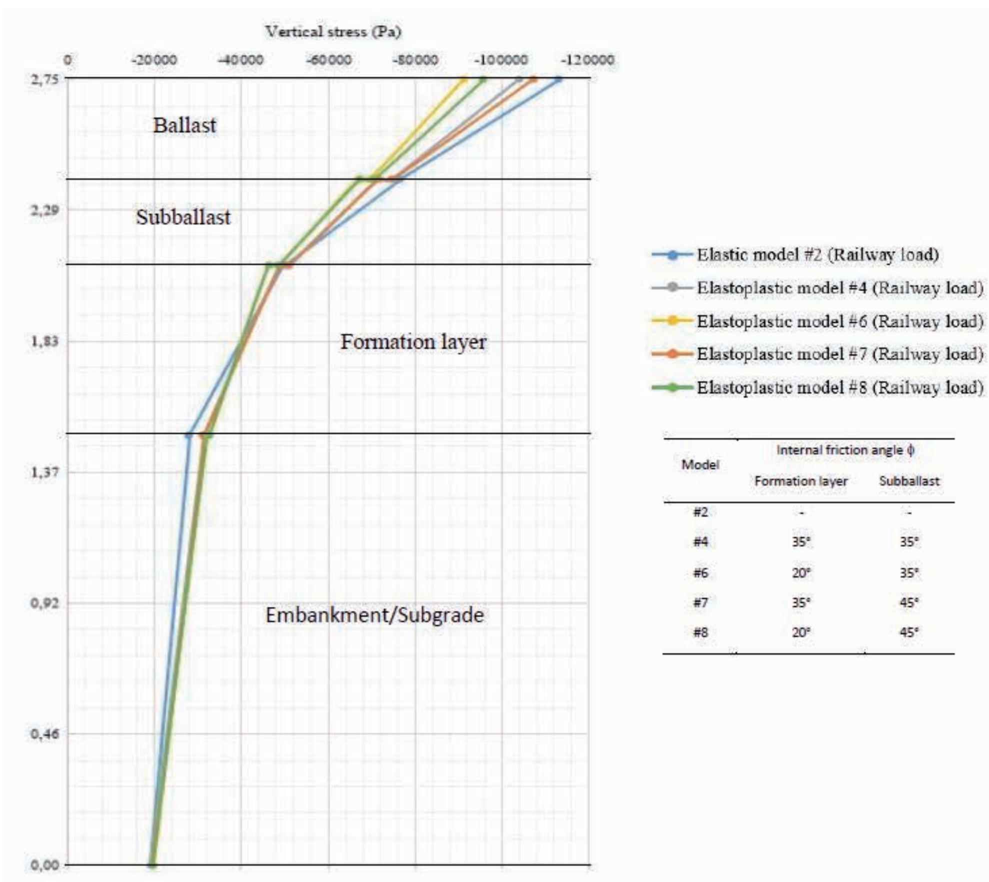

Fig. 7. Profile of vertical stresses due to the application of the railway load for all the models studied.

5. Conclusions

From the point of view of numerical modelling, it is appropriate to model the materials of the superstructure with a linear, homogeneous and isotropic elastic behaviour as these have a high modulus of elasticity compared to that of the granular layers. On the other hand, it is more advisable to carry out analyses that consider the elastoplastic behaviour of the materials, especially if they are granular, since the remaining or irreversible deformations are important because they can cause seats that can affect the longitudinal and transversal profile of the track.

As for the results obtained with the numerical model we can clarify that regarding the use of behavioural models. The use of an elastoplastic model leads to higher values of displacement due to the appearance of the plastic component of deformation besides that the use of an elastic model leads to an overestimation of the tensions with respect to those that would have in reality if an elastoplastic behaviour were assumed.

Finally, based on the analysis of all the previous results, some design recommendations can be used that can be used by the designer in the design of the railway platform:

1. If the Embankment or Subgrade has a high height, it is advisable to use an elastoplastic behaviour model due to the high numerical error that would be in the calculation of seats when considering an elastic model. If it has a smaller height, it is useful to use a model of elastic behaviour and. more if Formation layer is not available.
2. With poor materials or low resistance it is advisable to use an elastoplastic analysis against an elastic one due to the influence that would have in terms of seats and tensions, the layers of the infrastructure in their joint interaction.
3. The use of an elastic model will lead to an underestimation of seats that would be in reality, assuming that the railroad is more rigid than it really is.
4. Increasing the angle of friction of the Subballast from 35° to 45° does not imply any relevant changes in the values of the tensions and vertical seats for the rest of materials.

6. Notation

The following symbols are used in this paper	
H' = hardening parameter	ε^p = plastic deformation
E^{ep} = modulus of elastoplastic deformation	K = vertical track stiffness
E = modulus of elasticity	$\{\sigma\}$ = stress vector
c = material cohesion	{ } = strain vector
Φ = internal friction angle	[D] = constitutive matrix
ρ_i = vertical compression of each layer	[D^e] = elastic constitutive matrix
ε = deformation of the element	[D^p] = plastic constitutive matrix
ε^e = elastic deformation	[D´] = elastoplastic constitutive matrix

7. References

- ABU SAYEED, M. and SHAHIN, M.A. (2016). *Three-dimensional numerical modelling of ballasted railway track foundations for High-Speed trains with special reference to critical speed. J Tran*sp Geotech 6(8), pp. 55-65.
- ADIF (Administrador de Infraestructuras Ferroviarias). (2006). *Instrucciones y Recomendaciones para redaccion de Proyectos de plataforma-Geotecnia y Obras de tierra.* Madrid, Spain.
- ANSYS INC. (2008). ANSYS user´s manual. Canonsburg. PA. USA.
- ANSYS INC. (2013). ANSYS Mechanical APDL Theory Reference .Canonsburg. PA. USA.
- CHAVES, E.W.V. (2013). *Notes on Continuum Mechanics. International Center for Numerical Methods in Engineering (CIMNE),* Barcelona, Spain.
- FORT, L. and FORT, C. (2016). *España y la red de alta velocidad en Estados Unidos.* Revista de Obras Públicas 163(3580), pp. 36-53.
- GALLEGO, I. (2012). *Heterogeneidad resistente de las vías de alta velocidad: Transición terraplénestructura.* Editorial Académica Española, Madrid, Spain.
- GALLEGO, I., MUÑOZ, J., SANCHEZ-CAMBRONERO, S., RIVAS, A. (2013). *Recommendations for Numerical Rail Substructure. Modeling Considering Nonlinear Elastic Behavior.* J Transp Eng 139(8), pp. 848–858.
- GENS, A. and POTTS, D.M. (1988). *Modelos elastoplásticos de estado crítico para análisis numéricos de problemas geotécnicos. I- Formulación básica y principales modificaciones*. Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería 4, pp. 497-522.
- GOMES CORREIRA, A., WINTER, M.G., PUPPALA, A.J. (2016). *A review of suitainable approaches in transport infrastructure geotechnics*. J Transp Geotech 7(12), pp. 21-28.
- HUANG, Y.H. and TUTUMLUER, E. (2011). *Discrete element modeling for fouled railroad ballast.* J Constr Build Mater. 25(20), pp. 3306-12.
- INDRARATNA, B. (2016). 1st Proctor Lecture of ISSMGE: *Railroad performance with special reference to ballast and substructure characteristics.* J Transp Geotech 7(4), pp. 74-114.
- ISHIKAWA, T., MIURA, S., SEKINE.,E. (2014). *Simple plastic deformation analysis of ballasted track under repeated moving-wheel loads by cumulative damage model.* J Transp Geotech 1(4), pp. 157-170.
- JIMÉNEZ SALAS, J.A. (1981). Geotecnia y Cimientos II. Rueda, Madrid, Spain.
- JIMÉNEZ SALAS, J.A. (1980). Geotecnia y Cimientos III. Rueda, Madrid, Spain.
- LI, D., HYSLIP, J., SUSSMANN, T., CHRISMER, S. (2016). *Railway Geotechnics.* CRC Press/ Taylor & Francis Group, Boca Raton, FL, USA.
- LÓPEZ PITA, A. (2001). *La rigidez vertical de la vía y el deterioro de las líneas de Alta Velocidad.* Revista de Obras Públicas 148(3415), pp. 7-26.
- MCWILLIAMS, P., FERNÁNDEZ MERODO, J.A., PASTOR PÉREZ, M., NASARRE GOICOECHEA, J., CARRILLO, J. (2000). *Aplicaciones del Método de Elementos Finitos a la ingeniería ferroviaria.* Revista de Ingeniería Civil 118, pp. 71-82.
- MINISTERIO DE FOMENTO. (1999). *Recomendaciones para el proyecto de plataformas ferroviarias*. Madrid, Spain.

- OLIVER, X. and AGELET, C. (2002). Mecánica de medios continuos para ingenieros. UPC publications, Barcelona, Spain.
- POULOS, H.G. and DAVIS, E.H. (1974). *Elastic Solutions for Soil and Rock Mechanics.* John Wiley and Sons, New York, USA.
- SHIH, J.Y., THOMPSON, D.J., ZERVOS. A. (2017). *The influence of soil nonlinear properties on the track/ground vibration induced by trains running on soft ground.* J Transp Geotech 1(11). pp. 116.
- WINKLER, E. (1867). *Die Lehre von Elastizität und Festigkeit.* Prague.
- WOODWARD, P.K., KENNEDY, J., LAGHROUCHE, O., CONNOLLY, D.P., MEDERO, G. (2014). *Study of railway track stiffness modification by polyurethane reinforcement of the ballast.* J Transp Geotech 1(4), pp. 214-224.

8. Acknowledgements

Author would like to acknowledge the support of teachers and staff of the Department of civil engineering and building in the School of Civil Engineering of the University of Castilla-La Mancha as well as the Enrique Castillo Institute for the financial support necessary for the realization of this work.