

## Development and application of energy optimization models in a railway environment: from railway design to ecodriving

### *Desarrollo y aplicación de los modelos de optimización de energía en entornos ferroviarios: del diseño ferroviario a la conducción eficiente*

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#### Resumen

Este documento tiene por objeto describir y analizar algunas de las características y los resultados relativos al desarrollo y aplicación de nuevos modelos matemáticos concebidos para la optimización de funcionamiento de perfiles de velocidad, así como la energía de tracción consumida en la operación ferroviaria. En primer lugar se muestran los principios básicos y las fortalezas de los modelos para posteriormente discutir la importancia de la utilización de diferentes modelos matemáticos en todo el proceso de optimización de la energía mostrando los resultados de algunos ensayos ya realizados.

Finalmente se muestran importantes conclusiones derivadas de los potenciales ahorros en el consumo de energía, especialmente para los diferentes tipos de material rodante, condiciones de infraestructura y horarios.

Palabras clave: Optimización, algoritmo, conducción eficiente, energía, ferrocarril.

#### Abstract

*This paper aims to describe and to analyze some of the characteristics and results concerning the development and application of new mathematical models conceived for the optimization of running speed profiles as well as the traction energy consumed in the railway operation. First we show the basic principles and strengths of the models and afterwards discuss the importance of the use of different mathematical models in the whole process of energy optimization showing the results of some trials already done. In the swifter models we emphasize their potential in anin-cab optimization use which gives them ecodriving potential.*

*Finally important conclusions are drawn regarding the potential savings in energy consumption, especially for different types of rolling stock, infrastructure conditions and schedules.*

*keywords: Optimization, algorithm, ecodriving, energy, railway.*

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## **1. Introducción**

For some time now there has been a huge concern for energy efficiency among railway operators and many other stakeholders. Nowadays since the dramatic increase of the oil prices the energy problems have taken new proportions and the worries about the environmental consequences of poor management in natural resources are more present. In the past few years, cumulated experience from all over the world has been very helpful and now several practical and theoretical approaches concerning energy optimization in the railway sector can be easily identified. Simultaneously the introduction of new technologies, particularly in the rolling stock, allowed a faster materialization of some theoretical approaches with good results.

Both theoretical and practical approaches have been used with the purpose of finding new methods to optimize the speed profiles of the trains. In the last decades many theoretical solutions were developed based on different kind of algorithms particularly in metaheuristics with most of them having achieved quite good results. However some of those theoretical solutions are yet not tested from the operating point of view and so there are doubts concerning their performance. On the other hand, after knowing the general ideas of a reasonable optimized energy driving, more practical solutions have been introduced allowing for some interesting results mainly focused on the driver's on-site training. Yet, however interesting these practical approaches still lose a huge part of the potential for energy optimization.

The two models presented in this paper were developed with the sole purpose of optimizing the running speed profiles. The first one is based on linear programming and gives the optimal solution considering the infrastructure geometry, rolling stock characteristics and all the operational restrictions. The second approach resulted in a heuristic which is able to achieve a very good solution while being extremely fast. Thus the second model may be used in real time driving as it's capable to swiftly perform optimized speed profiles and still respect all the restrictions of the infrastructures, operation targets and the rolling stock characteristics (ecodriving).

### **Mathematical Models**

The main driving options of train which lead to a more rational energy consumption are well known: coasting after achieving the highest points; avoid mechanical braking; start up with the maximum power; cut the power mainly at the highest speeds. However it's extremely difficult for the driver to identify the geometry of the track profile while driving and so to know the exact locations where to change throttle position. Furthermore most often the driver doesn't know the impact of its driving options on the schedule hence he usually adopts a conservative approach considerably restricting the optimization potential of the run. This sort of situations clearly illustrate the relevance of the effort put into the developing of theoretical models to allow the identification of optimized drives that ensure schedule fulfilment and, simultaneously, consider all the infrastructure restrictions.

When formulating mathematical models for energy optimization it is very important to adequately identify the factors which can explain the traction consumption in one railway journey in order for them to be properly considered. Whereas energy optimization is concerned most mathematic formulations found often consider the geometric parameters of the railway, the operation restrictions or the major characteristics of the rolling stock. Over and over again we also see that authors use detailed information in their models such as the rolling stock length (Ko et al, 2004), the energy recovered on a regenerative braking systems or the energy consumed on the auxiliary equipment. Nevertheless the important is that mathematical model considers the inputs which are necessary to ensure the goals.

Although both part of an extensive process of optimization the two developed models presented in this paper assume different philosophies. The first one is called ECOLP (Energy Consumption Optimization Linear Programming) and uses a linear programming approach which allows us to find the optimal solution. When conceiving the goal was to develop a model that could give us the best possible solution considering the infrastructure and the rolling stock restrictions without any driving modes constrain. Hence in this model considers assumptions such as: the driver can increase or decrease power as often as he wants even when humanly impossible. This degree of freedom could lead us to unpractical solutions but at the same time ensures that the best solution we could ever achieve is found. In the equations (1) and (2) we can see in a very simplified way the linear programming approach formulation. The objective is to minimize the energy consumed (E) without jeopardizing the established journey time (T) and respecting all the speed restrictions (V) which are a function of the position of the train and its characteristics also speed related ( $f(x,V)$ ).

$$\text{Objective Function: } \min E \quad (1)$$

$$\text{Restrictions: } T \leq T_{\max} \quad \wedge \quad V(x) \leq f(x,V) \quad (2)$$

The approach integrates the basic concepts of kinematics and follows Newton's Second Law of Motion. As can be seen in the following equations, it was adopted the formulation with the position as an independent variable (in opposition to time as an independent variable). Consequently the speed of a train in a certain (x+1) position is calculated concerning the value of the speed in the position immediately before ( $V(x)$ ) and the acceleration or braking capacity ( $a(x)$ ) of the rolling stock. To calculate the acceleration we also need to know the total mass of the rolling stock (m) as well as the resistance forces of the train. The resistance forces of a train ( $F_r$ ) comprise foremost the wind resistance and the tangential part of the gravitational force ( $g_{tag}$ ). Knowing the rolling stock traction force ( $F_t$ ) we are able to calculate in every single iteration the total force and thus the value of the acceleration. With the values of the traction force, the mass of the rolling stock and the engine yields (g) we can easily calculate the energy (E).

$$V_{(x+1)}^2 = V_x^2 + 2 \cdot a_{(x,v)} \cdot \Delta x \quad (3)$$

$$a_{x,v} = \frac{F_{t(x,v)}}{m} + \frac{F_{r(v)}}{m} - g_{tag(x)} \quad (4)$$

$$t = \sum_{x=0}^{x-1} \left( \frac{\Delta x}{\frac{v(x) + v_{(x+1)}}{2}} \right) \quad (5)$$

$$E_x = \begin{cases} \frac{F_{t(x,v)} \cdot \Delta x}{\gamma_t} \quad \text{se } F_{t(x,v)} \geq 0 \\ F_{t(x,v)} \cdot \Delta x \cdot \gamma_f \quad \text{se } F_{t(x,v)} \leq 0 \end{cases} \quad (6)$$

The geometry of the railways is accurately deemed as well as the rolling stock characteristics. Like most of other mathematical models, the trains are considered as a single point and thus its length isn't taken into account. The energy is the product of the traction force and the distance, times the engine yield (6). When the rolling stock has a regenerative braking mechanism the ECOLP model is able to calculate the returned energy using a different yield value ( $g_f$ ). Nonetheless, regenerative braking is only considered till a certain value which can be defined in the model and establish the difference between the electric and the mechanical break.

Since the ECOLP model uses a linear programming approach we had to overcome the obstacle of the nonlinear equations (3), (4) and (5). To deal with this we used two linearization techniques: i) variable substitution; and ii) piecewise approximation. Although these techniques have some well-known mathematical limitations, they have proved to be quite effective in solving this sort of problems. In its final formulation the model was set up as a multi objective problem solver. The elementary objective function (1) was changed to a general scope function translated by formula (7). The latest includes coefficients related to the cost of energy and time ( $c_E$  and  $c_t$ ). With this option we have developed a model that is able to perform a natural trade-off between the energy consumption optimization and the journey time increase. Consequently a general objective function was reached comprising information about the auxiliary energy consumption, such as cooling or ventilation devices, losses of incomes due to the demand elasticity concerning the travel time, or the reduction of positive economic externalities benefits.

$$\text{Objective Function: } \min (c_E \cdot E + c_t \cdot T) \quad (7)$$

This exact model can then find the optimal speed profile for any journey and thus establish a maximum standard for each case. Therefore it's possible to quantify and analyze the differences between the ECOLP solution and the results of alternative approximated models.

After developing a mathematical model able to find the optimal solution we have looked forward to find a more simplified model. The goal was to ensure good and fast optimized solutions and simultaneously their operational and driving feasibility. This type of problem has been analyzed for many years using heuristics based on dynamic programming (Koet *et al*, 2004), genetic evolutionary stochastic algorithms (Chang and Sim, 1997; Bo and Wu, 2007; Wei *et al*, 2009) or swarm intelligence algorithms (Hu *et al*, 2010) and other metaheuristics. Although these models are able to reach good solutions they are usually complex, stochastic and many times computationally slow. There is then room to develop new models able to achieve faster optimized and deterministic solutions without compromising all kinds of operational restrictions.

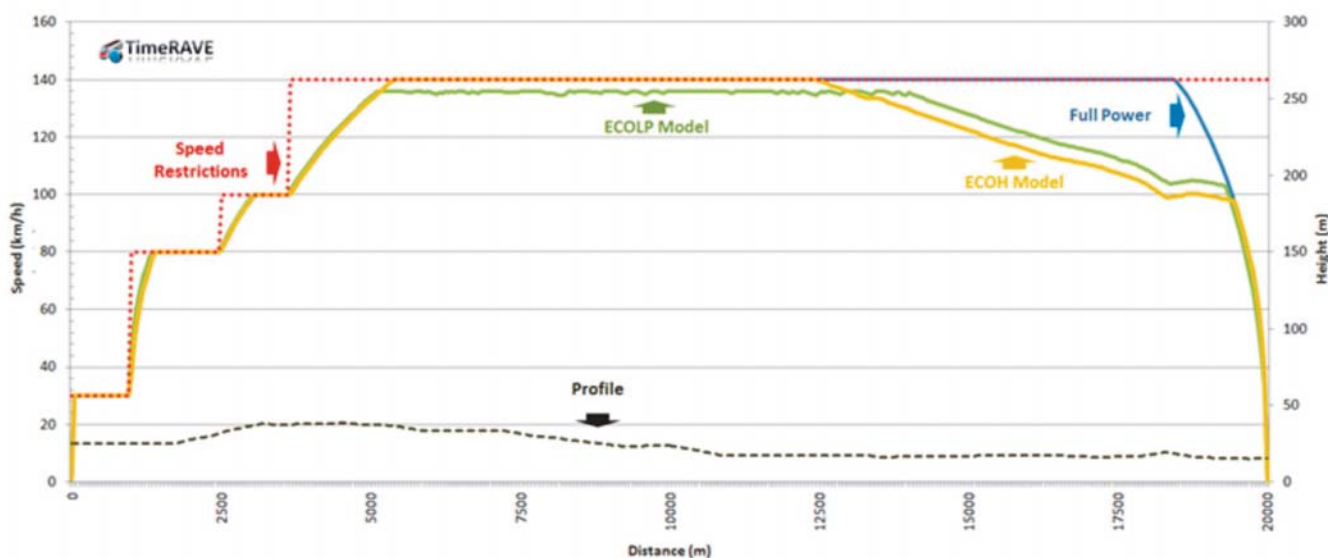
Guided by quality, simplicity and feasibility principles a new model called ECOH (Energy Consumption Optimization Heuristic) was developed. As the ECOLP model, the ECOH approach integrates the basic concepts of kinematics and follows Newton's Second Law of Motion as well. So the speed and energy calculation is based in the same simple equations (3), (4), (5), and (6) using an iterative process with distance increments.

The main philosophy of the ECOH approach is to find the best points where to start coasting or to maintain speed. Thus the calculation becomes simpler than the ECOLP model ensuring at the same time its consistency with real train driving. The ECOH approach is a heuristic with several sequential routines:

calculation of the minimum journey time in full power driving; identification of the section where the train must start coasting; identification of the sections where the train must maintain the speed. These routines are perfectly consistent with railway driving which is also directly with the rolling stock characteristics. Being a quite good representation of the operational reality the model only allows five different kinds of driving: full power (maximum acceleration); constant speed; coasting; regenerative braking; mechanical braking. With this approach the computation time is extremely shortened and the results match perfectly with real driving and operation options as well as restrictions.

## 2. Model's performance

The models were tested and validated under different perspectives in order to show their performance, feasibility and suitability to real life situations. Two approaches were followed: detailed analysis of ECOLP model running time and results; comparative analysis of ECOH model faced to ECOLP model, in order to assess the quality and robustness of the results of the former model (Talbi, 2009). Simultaneously the evaluation of the differences between theoretical results and real operational requirements were compared. In all measurements done in real journeys we have tried as far as we could to quantify the energy consumption and thus the potential of the energy saving given by the models.



**Figure 1.** Optimized journeys using the ECOLP and ECOH models

In order to illustrate the behaviors of the two developed models we shall analyze the speed profiles for a particular railway stretch (Figure 1). In yellow we can see the ECOH speed profile and in green the ECOLP profile. Figure 1 shows that from the real driving point of view the ECOLP model gives us a quite feasible result even without any driving restriction in its mathematical approach. The only exception can be easily identified in Figure 1 as it occurs in a section where the proposed driving is absolutely impossible to perform, approximately between point 5000m and 14000m. In this section the ECOLP model presents several humanely and technically impossible changes in the speed. To correct this situation the ECOH model suggests the implementation of a constant speed slightly higher than the ECOLP model and starts coasting



earlier. When we have compared the values of the traction energy consumption given by the two models we realize that they are almost alike. This sort of result was found in most of the analysis performed for different types of railway services and rolling stock. Such results allow us to reach the following conclusion; besides returning the absolute maximum value of energy saving, for most cases the ECOLP model is also able to offer, feasible driving solutions.

The second step was to evaluate the performance and the quality of the ECOH model results. Comparing the results between the heuristic and the linear programming approach (ECOLP) we have concluded that their values were very similar. In average the difference among the two theoretical models was under 2% ( $1,00 < E_{\text{ECOH}} / E_{\text{ECOLP}} < 1,02$ ). Furthermore we found that the differences between the two models were only slightly larger only in two specific situations. The first situation is when we increase the journey time. However only in unusual situations such as when the difference to the full power journey time was higher than 30% did the results show a slight diversion. The second situation occurred mainly in suburban movements where the maximum speed is not too high. Still the results of both models were always very close. From the computational point of view the ECOH model has revealed to be extremely fast even for quite long stretches. This property enables several swift and consecutive energy optimizations making it possible to be used in an in-cab application (ecodriving).

### 3. Real-life Tests

Comparing the theoretical results using the mathematical models with the values of the real time journeys we have clearly identified several situations where it's possible to achieve very significant energy reductions (see Table 1). This type of situation remains truth even in scenarios where the operator has already taken measures in order to reduce traction energy consumption. In our tests we used different railway services with different types of rolling stock and in all situations we found great potential savings. The major potential savings were found in long distance services. In these cases we easily found traction energy savings over than 20% comparing to present situations. Although also promising, the results for the suburban services were not so impressive. This mainly happened because the schedules of the suburban services analysed had less freedom. Thus the trains were operating most of the times in full power condition. Yet there were several cases of suburban services where we have achieved greater potential savings without sacrificing the established schedules.

| Scenario           | Journey Time |          | Average Speed |          | Total Length |          | Consumed Energy |          |
|--------------------|--------------|----------|---------------|----------|--------------|----------|-----------------|----------|
|                    | Intercity    | Suburban | Intercity     | Suburban | Intercity    | Suburban | Intercity       | Suburban |
| Full Power Journey | 87.0 min.    | 5.0 min. | 145 km/h      | 106 km/h | 211 km       | 9 km     | 1794 kWh        | 67 kWh   |
| Performed Journey  | 92.8 min.    | 6.2 min. | 136 km/h      | 87 km/h  | 211 km       | 9 km     | 1806 kWh        | 52 kWh   |
| Optimized Journey  | 95.5 min     | 6.0 min. | 132 km/h      | 90 km/h  | 211 km       | 9 km     | 1340 kWh        | 42 kWh   |

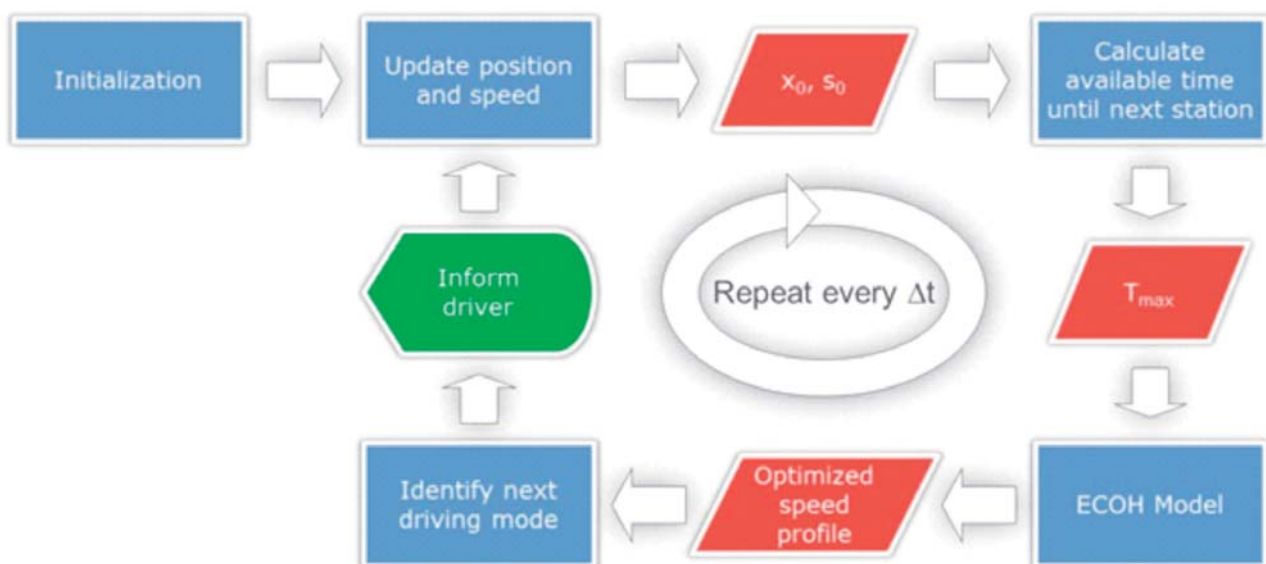
**Table 1.** Real measurements and optimized journeys using the ECOH approach

Although we haven't got real measurements concerning freight services so far, our expectations are very optimistic. Several theoretical testes were undertaken in order to compare full powerjourneys for freight trains and optimize speed profiles. The results have clearly shown a significant energy saving in this kind of service with small time increments on the established schedules. It is our opinion is that this happen especiallyfortwo mainreasons: first in these casesthere issignificant energy consumption as a direct consequence of the cargo weight; secondwe usually find more permissive schedules, hence withmorespacefor optimization. In short, we found evidences that there is a significant portion of energy waste in freight services too. These circumstances could be easily solved with the theoretical approaches exposed in this paper.

#### 4. Ecodriving

Based on the ECOH model presented in previous sections, a prototype of a real-time driver assistant system (Ecodriving) was developed. The system architecture, supported by GPS technology, allows it to work stand alone. The goal behind this architecture is to swiftly performoptimized speed profiles regarding the established schedules without the need to communicate with any train instrument. This way the system can give periodic instructionsabout the more economic driving options to the driverwithout compromisingany safety condition.

The developed system uses dynamic optimization routines which continuously updates both trainposition and speed recalculating the optimized speed profile, as illustrated in the next figure.



**Figure 2.** Flowchart of Dynamic Optimization Routines embedded in Ecodriving System

In addition to suggesting the best driving mode at each moment, the system is capable to provide other useful information to the driver. For instance it may also offer the next recommended driving mode along with where and when it will occur, the current and next speed limits, the time and distance to the next

station and to the end of journey, the compliance with the schedule, the driving energy efficiency and so forth. The driver may enhance the information he wants to pay more attention thus ensuring the highest effectiveness of the system.

The prototype is now in an on board testing period. The first tests supported the aptitude of the system to be implemented as an in-cab tool since the optimization algorithm remains extremely fast even in long railways routes. The system also proved to be very efficient in adjusting to perturbations during the journey. Furthermore it was shown that the inputs possibilities of the ECOH model cover the majority of the operational situations such as the rolling stock characteristics, signalling or infrastructure specificities.

## **5. Potential applications**

Several important applications of the developed models (ECOH and ECOLP) are foreseen both for operators and railway infrastructure managers. Although most of situations are frequently connected to operator's business, the models may also be used in a wide range of situations by railway infrastructure managers. For instance, in early stages of studies development (feasibility or design) of a future railway line it's possible to analyse and optimize technical options while ensuring its operation feasibility. Consequently, by using this sort of tools and methodologies it's possible to achieve construction cost reductions even in the preliminary steps of the studies. Another possible optimization using these mathematical approaches is the railway schedules where it is possible to achieve the best trade-off between established journey times and the energy consumptions.

For the railway operators, however, it's possible to identify more situations where the developed tools may be used. In fact, there are several perspectives from which these approaches can be quite profitable to an operator. Between all the possible benefits the most important one is the direct consequence in energy consumption of an intelligent driving, which is none the less but the energy bill reduced. As mentioned before, this kind of tool can identify optimized speed profiles of a certain railway stretch. So, for every available schedule the driver is able to perform an optimized journey by knowing exactly where to accelerate, coasting or breaking. Besides if a real-time driving assistant is installed in the train the driver can perform an optimized driving even if changes occur during the journey. Comparing to present situations, results have shown that with these methodologies it's easy to achieve energy saving over 20%.

Another possible use to this type of tool may be to train and to evaluate drivers. The identification of optimized journeys increases the driver's knowledge about ecodriving since it establishes several relations between the speed profile, the infrastructure and the rolling stock own characteristics. Moreover is possible to use this sort of theoretical approach to stimulate a more rational conscience among the drivers. At the same time the successes and the failures of the drivers could be evaluated by establishing a balance between a real journey and the benchmark, generated by the ECOLP model.

It is also possible to quantify any disturbances on the railway operation using ECOH and ECOLP models. Every change on the schedule can be evaluated. For example we can quantify the extra energy consumed in a delay due to an infrastructure work. But even in these situations is possible to mitigate the energy consumption and optimize the journey with different restrictions and goals. This type of evaluation is useful for both the railway operators and the infrastructure managers.



## **6. Conclusions**

Two different mathematical models were developed, named ECOLP and ECOH. Both models can easily generate optimized speed profiles considering the infrastructure geometry, rolling stock characteristics and all the operational restrictions. However they have different goals in the energy optimization process and as we have shown the two methods complement each other.

The ECOLP model is able to find the optimal solution but its computational time is high. Furthermore the ECOLP model results have shown that although the model doesn't integrate any driving constraint, the solutions were most of the times feasible from the driver's point of view. These characteristics have revealed that this model could be a very strong tool to solve planning and design problems, such as schedules or technical alignments optimization. In the other hand the ECOH model has demonstrated a very fast computational time and with optimized solutions very close to the optimal. The solutions are also absolutely adjustable to realtime train driving. For these reasons we claim that the ECOH model could be a powerful tool to be used in all situations especially in an in-cab driver assistant system (ecodriving).

Several real-life tests were done and some of them are presented in this paper. The results clearly show that using these two approaches it's possible to achieve an energy saving over 20% for the train operators. Besides the methodologies work very well in all kind of trains and services. We have done several experiences in suburban railway connections as well as in long distance trains with very expressive results. Although only theoretical tests were made concerning freight trains services, reached results revealed considerable energy savings too.

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